I ſhall conclude with my earneſt entreaty, that my moſt humble ſervice may be preſented to the Noble Members of the Royal Society, and remain

Honour'd Sir, *Your Humble Servant,*
Anthony Van Leeuwenhoek.

IV. *Reverendi D.* Johannis Craig, *Epiſtola ad Editorem continens ſolutionem duorum problematum.*

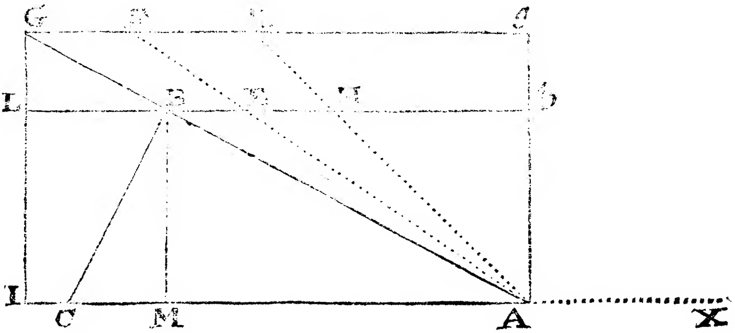
Ad Eruditiſſimum Virum Dominum *H. Sloane*, M. D. & R. S. Secretarium.

Mitto tibi, vir clariſſime, ſolutiones, duorum Problematum; quibus ſolvendis operam dederunt (& etiamnum dant) Celeberrimi hujus ætatis Mathematici. Prius eſt de inveniendo Solido Rotundo, quod minimam in fluido patiatur reſiſtentiam, ab incomparabili viro D. Iſ. Newtono *jam olim ſolutum; quod denuo nuper aggreſſi ſunt, Illuſtriſſimus Marchio Hoſpitalius, & Celeber.* Jo. Bernoulli, *ulterius exponere; quoniam Analyſin ſuam ſupprimere voluit Digniſſimus* Newtonus. *Poſterius autem Problema eſt de inveniendâ Lineâ celerrimi deſcensûs; quod ante hos quatuor annos omnibus (ut noſti) Europa Mathematicis à clariſſ.* Jo. Bernoulli *proponebatur, & jam ſæpius ſolutum fuit. Ad meas ſolutiones quod attinet: Eas jam publici juris facio (non quòd me quicquam magni momenti præclaris eorum laboribus addere poſſe ſperem, ſed) ut majori eaſdem res tractandi varietate, ad majora Scientiæ illæ incrementa promoveantur. Et quamvis ſeriùs prodeat mea de Curvâ celerrimi deſcensus Analyſis; magnâ tamen ejus ſimplicitate mora (ut ſpero) compenſabitur. Qualem alii adhibuerint, neſcio; cum nulla hujus ſolutio (nec quæ in veſtris, nec quæ in Leipſicis Actis eduntur) ad manus meas adhuc pervenerit, præter* Newtonianam, *quæ Analyſin, non exhibet. Si inter ſelectas tuas Collectiones Philoſiphicas, tenues etiam hæ noſtræ loco aliquo dignæ videantur, habebis tibi devinctiſſimum,*

Gillingham, 21 *Dec.* 1700.

JO. CRAIG

Lemma. INvenire rationem inter reſiſtentiam, quam patitur Triangulum rectangulum AIG,& reſiſtentiam quam patitur rectangulum circumſcriptum AIGg dum utrumque in fluido movetur juxta directionem Lineæ IA, ab I verſus X.



A puncto quovis B ducantur BC normalis ad AG ; & Bb parallela ad AI, item BM normalis ad AI. Tum in bB capiantur bH=$\frac{CM^2}{BC}$ & bE=BC ; & per puncta H, E ducantur rectæ HA, EA, quæ productæ ſecent Gg in K & F : Dico Reſiſtentiam Trianguli AIG eſſe ad reſiſtentiam Rectanguli AIGg ut Area trianguli AKG, ad Aream Trianguli AFg. Imo & reſiſtentiam in partem quamlibet lineæ AG ad reſiſtentiam in partem correſpondentem lineæ Ag exem. gra. in AB & Ab ut Area AHB ad Aream AEB. Demonſtratio pendet à Theoremate generali, quod facillimè deduxi ex Prop. xxxv. Newtoni, p. 324.

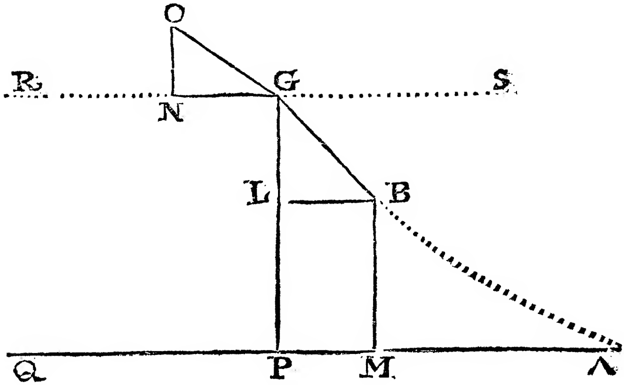
Corol. 1. Sint jam BG,bg partes infinitæ parvæ linearum AG, Ag,& producatur bB ad L ; dico reſiſtentiam in BG (quam vocemus e) eſſe ad reſiſtentiam in bg (quam vocemus E) ut GL^2 ad GB^2.

Nam e. E :: KHbg. FEbg, id eſt e E :: bgxbH. bgxbE (per Lemma præcedens) Ergo e. E :: bH. bE, id eſt e E :: $\frac{CM^2}{BC}$ BC (per conſtructionem ſuperioris Lemmatis) Ergo e. E :: CM^2. BC^2. Sed CM^2. BC^2 :: GL^2. GB^2. (ob ſimilia Triangula BMC, GLB) Ergo e. E :: GL^2. GB^2. Q. e. D.

Corol. 2

Corol. 2. Reſiſtentia in partem infinite parvam GB eſt æqualis Cubo lineæ GL diviſo per Quadratum lineæ GB. Nam ſi omnes partes infinitè parvæ in linea Ag ut bg ſupponantur æquales, tum Reſiſtentia in bg per ipſam bg exprimi poſſit, id eſt, $E = bg$, adeoque $E = GL$ Ergo per Corollarium primum $e . GL :: GL^2 . GB^2$; unde $e = \frac{GL^3}{GB^2}$ Q. e. D.

Corol. 3. Sit r radius & c circumferentia cujuſvis circuli, dico reſiſtentiam in conicam ſuperficiem genitam à rotatione, lineolæ GB circa AI eſſe æqualem producto ex $\frac{C x BM}{r}$ in $\frac{GL^3}{GB^2}$ Nam reſiſtentia in Conicam illam ſuperficiem eſt æqualis omnibus reſiſtentiis in lineolam GB, id eſt omnibus e; id eſt æqualis circumferentiæ circuli cujus radius eſt BM in e multiplicatæ; id eſt, reſiſtentia in Conicam illam ſuperficiem eſt æqualis $\frac{c x BM}{r} x e$; adeoque per Corol. 2. æqualis $\frac{c x BM}{r} x \frac{GL^3}{GB^2}$ Q. e. D.

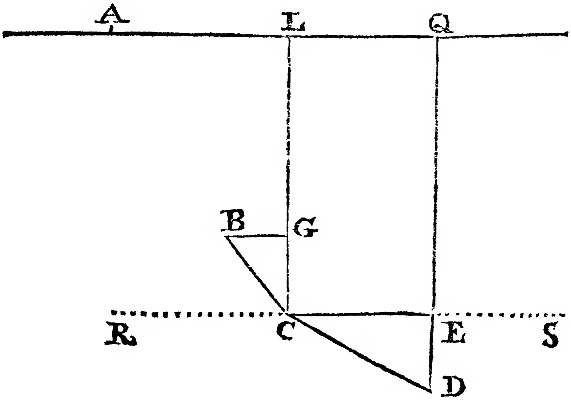


Problema 1. **Invenire Lineam curvam cujus rotatione producatur Solidum rotundum, quod (dum in medio fluido ſecundum axis ſui directionem movetur) minimam patiatur Reſiſtentiam.**

Sint OG, GB duæ particulæ infinitè parvæ in Curvâ quæſitâ, quæ circa AQ protata producat Solidum rotundum minimæ Reſiſtentiæ. Ducantur BM, GP normales ad AQ, item BL, GN ad AQ, & ON ad BM parallelæ: Jam $\frac{c \times BM \times GL^3}{r \times GB^2}$ eſt reſiſtentia in ſuperficiem genitam a rotatione lineolæ GB circa AQ, & $\frac{c \times GP \times ON^3}{r \times OG^2}$ eſt reſiſtentia in ſuperficiem genitam ſimiliter ab OG per *Cor.* 3. Jam utraque hæc Reſiſtentia ſimul ſumpta debet eſſe minima ſcil. $\frac{c \times BM \times GL^3}{r \times GB^2} + \frac{c \times GP \times ON_3}{r \times OG^2} =$ minimæ. Adeoque in linea RS ita ad AQ parallela ut ON ſit $=$ GL, quærendum eſt punctum G ut hoc contingat; quod ſupponendo puncta O & B eſſe fixa facile invenietur per notiſſimam Maximorum & minimorum Methodum. Calculum proſequendo devenietur tandem ad $\frac{BM \times BL}{BG^4} = \frac{GP \times NG}{OG^4}$; unde patet $\frac{BM \times BL}{BG^4} =$ conſtanti; ſic ſi abſciſſa AM vocetur x, & ordinata BM, y, erit $BL = dx$, $LG = dy$ (quam conſtantem in toto hoc calculo ſuppoſui) adeoque $BG^2 = dx_2 + dy_2$, unde $\frac{ydr}{dxdx + dydy^2} =$ conſtanti; Sit a linea quælibet conſtans & proinde, ut obſervetur Lex homogeneorum erit $\frac{ydx}{dxdx + dydy^2} = \frac{a}{dy_3}$ ut ab Illuſtriſſ. Hoſpitalio & celeber. *Jo. Bernouillio* inventum eſt. Et hic obiter clariſſ. Bernoullio ſignificare viſum eſt me plurimum delectari methodo ſuâ conſtruendi curvas ex æquationibus differentialibus, in quibus deeſt altera ex indeterminatis x vel y, in Actis Lipſicis publicatâ menſe Maio, Anni 1700. & per quam eleganter deduxit conſtructionem Curvæ modo quæſitæ. Nov. 1699. pag. 515.

Problema

Problema 2. Invenire Lineam Celerrimi Deſcenſus.



Sint BC, CD duæ particulæ infinitè parvæ in curva quæſita. Jam Curva illa debet eſſe talis ut tranſitus a B ad D poſt caſum a horizontali AQ fiat in tempore minimo; quærendum itaque eſt punctum in linea RS (ita ad AQ parallela ut differentiæ ordinatarum GC, DE ſint æquales) tale punctum C ut hoc contingat.

Jam velocitas ejus in puncto C eſt $\sqrt{LC}$ & velocitas in puncto D eſt $\sqrt{QD}$; Ergo $\frac{BC}{\sqrt{LC}}$ eſt tempus deſcenſus per BC, & etiam $\frac{CD}{\sqrt{QD}}$ eſt tempus deſcenſus per CD (per Prop. liv. pag. 158 Newtoni) Ergo punctum C debet eſſe tale ut $\frac{BC}{\sqrt{LC}}+\frac{CD}{\sqrt{QD}}$ = minimo. Supponendo B & D eſſe fixa, ſint conſtantes GC=DE=m, LC=b, QD=p; indeterminatæ BG=u, CE=z; unde $\frac{\sqrt{m^2+u^2}}{\sqrt{b}}+\frac{\sqrt{m^2+z^2}}{\sqrt{p}}$ = minimo; Ergo $\frac{udu}{b^{\frac{1}{2}}\sqrt{m^2+u^2}}+\frac{zdz}{p^{\frac{1}{2}}\sqrt{m^2+z^2}}=0$ ſed $du=-dz$ (quia $v+z=$ conſtanti) Ergo $\frac{u}{b^{\frac{1}{2}}\sqrt{m^2+u^2}}=\frac{z}{p^{\frac{1}{2}}\sqrt{m^2+z^2}}$; unde patet

$b^{\frac{1}{2}}\sqrt{m+}$

$\frac{u}{b^{\frac{1}{2}}\sqrt{m^2+u^2}}$ = conſtanti; ſit jam Abſciſſa $AL = x$; ordinata $LC = y$; adeoque $BG = dx$, $GC = dy$, $BC = \sqrt{dx^2}$; ſitque a linea quælibet conſtans Erit $\frac{dx}{y^{\frac{1}{2}}\sqrt{dx^2+dy^2}} = \frac{1}{\sqrt{a}}$, unde $dx\sqrt{a} = \sqrt{y} \times \sqrt{dx^2+dy^2}$. Sed in omni Curva dx eſt, ad $\sqrt{dx^2+dy^2}$ ut Subtangens ad Tangentem; Ergo talis eſt natura Curvæ quæſitæ ut ejus ſubtangens ſit ad Tangentem ut $\sqrt{a}$ ad $\sqrt{y}$. Quam utique Cycloidis proprietatem eſſe ſciunt omnes, quibus notum eſt Tangentem Cycloidis eſſe parallelam Chordæ arcus contermini in Circulo genitore, cujus Diameter eſt a, & cujus vertex deorſum ſpectat.

Et pari facilitate Curvam invenire poſſum Celerrimi Deſcenſus pro qualibet alia gravitatis Hypotheſi.